Proceedings of the Second Joint EMBS/BMES Conference
Houston, TX, USA • October 23-26, 2002

COMPUTING METRICS ON ANATOMICAL SHAPES IN COMPUTATIONAL ANATOMY

M. F. Beg[1], *M. I. Miller*[1], *A. Trouvé*[2], *L. Younes*[3]

[1]Center for Imaging Science, Johns Hopkins University, Baltimore MD 21218, USA.
[2]Université Paris 13, France.
[3]Ecole Normale Supérieure de Cachan, F-94 235 Cachan CEDEX, France

ABSTRACT

Metric distances can be used to quantify the notion of close and far on anatomical shapes as represented in images. This is achieved by computing diffeomorphic transformations between given images and measuring their size. Transformations that are "far" from identity represent larger differences in shape and size than those "close" to identity. Such metrics may find possible clinical applications such as the detection and study of shape and size changes in various diseases that manifest in shape and size changes of anatomical organs.

1. INTRODUCTION

In the deformable template model, the space of anatomical configurations $\mathcal{I}$ is an orbit under the group $\mathcal{G}$ of diffeomorphic transformations acting on the coordinate space Ω of the configurations. The fundamental assumption within this model, given two images I_0, I_1 in the orbit of a template, is that there exists an element of the diffeomorphisms φ_1 matching the images $I_1 = \varphi_1 I_0$. The diffeomorphism matching the images is the end point of a path $\phi : [0,1] \to \mathcal{G}$ in the space of diffeomorphisms which is generated from a time-dependent vector field $v : [0,1] \to V$ satisfying

$$\frac{\partial \phi_t}{\partial t} = v_t \circ \phi_t \quad \text{with} \quad \phi_0 = \text{Identity} \qquad (1)$$

where V is a Hilbert space of smooth vector fields. The notation ϕ_t^v is used to make explicit the dependence of ϕ_t with its associated velocity field v and $\phi_{s,t} : \Omega \to \Omega$ is used to denote the composition $\phi_{s,t} = \phi_t \circ (\phi_s)^{-1}$. The interpretation of $\phi_{s,t}(y)$ is that it is the position at time t of a particle that is at position y at time s. The metric distance on the space of images is defined by

$$\rho_{\mathcal{I}}(I_0, I_1) \doteq \inf \Big\{ \rho_{\mathcal{G}}(\text{Id}, \varphi_1) \mid I_1 = \varphi_1 I_0,\ \varphi_1 \in \mathcal{G} \Big\}. \qquad (2)$$

The metric distance $\rho_{\mathcal{G}}$ between two points in $\mathcal{G}$ is defined as

$$\rho_{\mathcal{G}}(\varphi_0, \varphi_1) \doteq \inf \Big\{ \int_0^1 \|v_t\|_V dt \mid \varphi_1 = \phi_1^v \circ \varphi_0 \Big\}, \qquad (3)$$

it is the shortest length path joining the points and is a distance on $\mathcal{G}$ for which $\mathcal{G}$ is complete. The inner-product $\langle\ ,\ \rangle_V$ is defined through a differential operator L on $C_c^\infty(\Omega, \mathbb{R}^n)$ such that $\langle u, v\rangle_V \doteq \langle Lu, Lv\rangle_2$ where $\langle\ ,\ \rangle_2$ is the usual L^2-product for square integrable vector fields on Ω and that V is the completion of $C_c^\infty(\Omega, \mathbb{R}^n)$ for the chosen inner-product. Details of this construction can be found in [1, 2, 3, 4].

The computation of the metric distance requires the estimation of matching diffeomorphism for the images φ_1 as well as the shortest length path connecting the identity element to the estimated φ_1 in the space $\mathcal{G}$. In the inexact matching setting, this estimation is achieved by minimizing an associated energy function

$$E(v) = \int_0^1 \|v_t\|_V^2 dt + \int_\Omega |I_0 \circ \phi_{t,0} - I_1|^2 dy \qquad (4)$$

where the first term is the regularization term and the second term is the data or the fitting term.

2. METHOD

Let I_0, I_1 be the template image at time $t = 0$ and the target image at time $t = 1$ of the flow respectively. The variational gradient of the energy functional $E(v)$

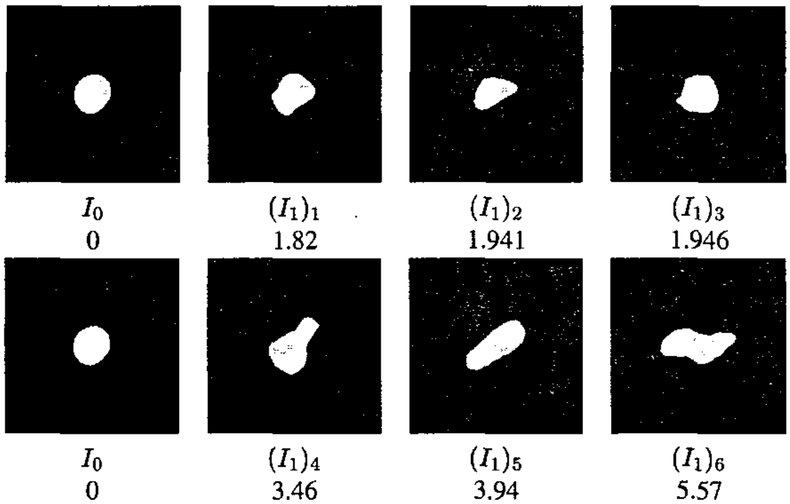

Fig. 1. Shown are computed metric geodesic distances between the template mitochondria image (left column) and other mitochondrial images.

in the space V is calculated to be [5] :

$$\nabla_v E_t = v_t - (L^{\dagger}L)^{-1}\Big(\frac{2}{\sigma^2}|D\phi^v_{t,1}|(J^0_t - J^1_t)\nabla J^0_t\Big) \quad (5)$$

where $J^0_t = I_0 \circ \phi^v_{t,0}$ and $J^1_t = I_1 \circ \phi^v_{t,1}$. This variational gradient is used in a standard gradient descent method [5] where $L \doteq -\alpha\nabla^2 + cI$ and for 2D, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

3. MAPPING MITOCHONDRIA IMAGES

We show results of computing metrics on a few simple shapes from biological images, namely mitochondria segmented from cell electron-microscopic images. The images are 64×64 pixels and were rigidly registered to remove rotation and translation. Operator $L = -0.01\nabla^2 + 1I$ and flow was discretized into 20 steps of size 0.1. Figure 1 shows the images as well as the corresponding metric distances calculated from the template image.

4. DISCUSSION

We present a method for computing metrics on anatomical images. Such metrics will provide useful information about the anatomical configurations contained in the images and may be useful in clinical detection, diagnosis and followup of various diseases that manifest in changes in shape and size of the anatomical configurations.

5. REFERENCES

[1] M.I. Miller and L. Younes, "Group actions, homeomorphisms, and matching: a general framework," *International Journal of Computer Vision*, vol. 41, pp. 61–84, 1999.

[2] P. Dupuis, U. Grenander, and M.I. Miller, "Variational problems on flows of diffeomorphisms for image matching," *Quarterly of Applied Mathematics*, vol. 56, pp. 587–600, 1998.

[3] L. Younes, "Computable elastic distances between shapes," *SIAM J. Applied Math.*, vol. 58, pp. 565–586, 1998.

[4] Alain Trouvé, "Diffeomorphic groups and pattern matching in image analysis," *Int. J. Computer Vision*, vol. 28, pp. 213–221, 1998.

[5] M. F. Beg, M. I. Miller, A. Trouvé, and L. Younes, "Computing metrics via geodesics on flows of diffeomorphisms," *In Preparation*, 2002.