Scalable Sparse Subspace Clustering by Orthogonal Matching Pursuit

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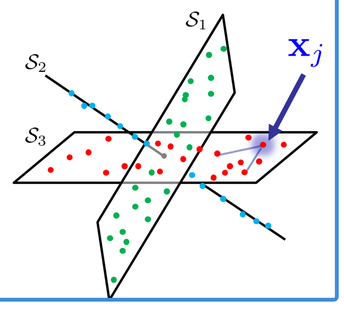
Motivation

- Vision datasets often contain multiple classes, each lying in a low-dimensional subspace
- **Subspace clustering**: cluster data that lie in a union of subspaces

Spectral Subspace Clustering

- Approach
  - Step 1: build data affinity
  - Step 2: apply spectral clustering
- Challenge: distance based affinity fails at the intersection of subspaces
- Solution: learn affinity by data self representation, i.e., $\mathbf{x}_j = X\mathbf{c}_j$, where $X = [\mathbf{x}_1, \ldots, \mathbf{x}_N]$



Sparse Subspace Clustering (SSC)

- Learn affinity by finding the sparsest data self representation

$$\min_{\mathbf{c}_j} \|\mathbf{c}_j\|_0 \quad \text{s.t.} \quad \mathbf{x}_j = X\mathbf{c}_j, \quad c_{jj} = 0$$

Prior Method

- Basis Pursuit (BP):
  - Replace $\|\mathbf{c}_j\|_0$ with $\|\mathbf{c}_j\|_1$
- Properties:

✓ Guaranteed correct connections under broad conditions

✗ Not scalable: solved by the CVX software or ADMM algorithm

Contribution

- Orthogonal Matching Pursuit (OMP):
  - Choose one point at a time
- Properties:

✓ Guaranteed correct connections under broad conditions

✓ Scalable: performance is verified on 100,000 data points

Guaranteed correct connections

Theorem

Suppose that $\mathbf{x}_j \in \mathcal{S}_\ell$. Then, $\mathbf{c}_j$ gives correct connections if

$$\mu(W_j^\ell, X^{-\ell}) < r^\ell$$

- μ captures the similarity between $\mathcal{S}_\ell$ and all other subspaces
- r captures the distribution of points in subspace $\mathcal{S}_\ell$
- W_j^ℓ is the dual points/residual points for SSC-BP/SSC-OMP

Theorem

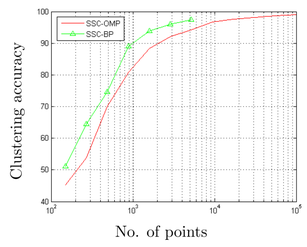
Randomly generate n subspaces of dimension d in ambient dimension D, and $\rho d + 1$ points from each subspace. $\{\mathbf{c}_j\}_{j=1}^N$ gives correct connections with high probability if

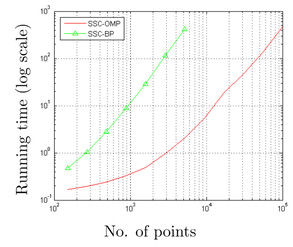
$$\frac{d}{D} < \frac{c^2(\rho)\log\rho}{12\log N}$$

- Same condition for SSC-BP/SSC-OMP, with different probability

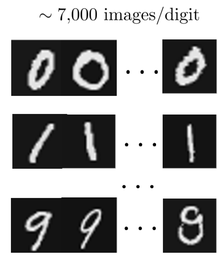
Experiments on Synthetic, Face Image and Digit Image Data

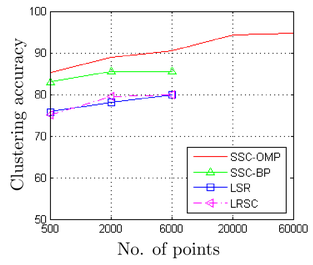
- Synthetic data: draw 5 subspaces of dimension 6 in ambient dimension 9; draw equal number of points from each subspace
- Clustering accuracy: SSC-OMP is slightly outperformed by SSC-BP, and the difference decreases as number of points increases
- Running time: SSC-OMP is orders of magnitude faster than SSC-BP, and is able to handle up to 100,000 points efficiently

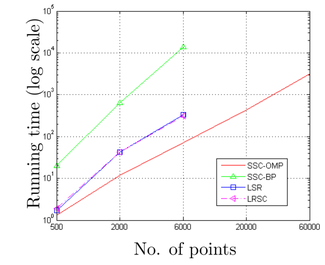




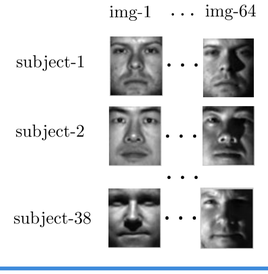
- MNIST handwritten digit database
- First time to test on 60,000 images
- Clustering accuracy: SSC-OMP obtains the best performance
- Running time: SSC-OMP can handle more points than other methods

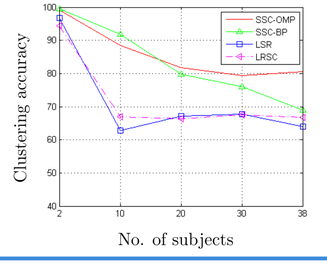


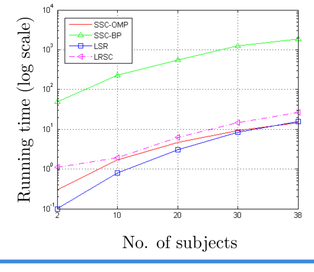




- Extended Yale B face database
- First time to test on all 38 subjects
- Clustering accuracy: SSC-OMP and SSC-BP achieves state of the art
- Running time: SSC-OMP is $>$ 100 times faster than SSC-BP







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