All problems count equally. You may use any result or formula mentioned in class. Carefully explain your answers.

**PROBLEM 1.** A fair die is rolled twice. Let \( X \) be the number of 2’s obtained and let \( Y \) be the number of 5’s obtained.

a) Find the joint distribution of \( X \) and \( Y \).

b) Are \( X, Y \) independent random variables?

c) Compute \( E[X + Y] \) and \( E[XY] \).

**PROBLEM 2.** The joint pdf of two random variables \( X \) and \( Y \) is \( f(x, y) = 8x^3y \) for \( 0 \leq x \leq 1 \) and \( 0 \leq y \leq 1 \).

a) Compute the marginal densities of \( X \) and \( Y \).

b) What is the correlation coefficient \( \rho \) between \( X \) and \( Y \)? Between \( X \) and \( 6 - X \)? No computation is necessary in either case but explain your answers.

c) Calculate \( P(XY \leq 0.5) \).

**PROBLEM 3.** Suppose a rare genetic defect occurs in 1 out of every 10,000 newborns in a certain country. If 25,000 babies are born in May of 2003 in this country, and the births are assumed independent of one another, use the Poisson distribution to estimate the probability of no genetic defects of this type among the 25,000 babies, and the probability of 3 or more incidences.

**PROBLEM 4.** The force \( X \) acting on a column to support a building is normally distributed with mean \( \mu = 20 \) kips.

a) If \( \sigma = 1.3 \) kips, compute \( P(21 \leq X) \).

b) What is the probability that \( X \) differs from \( \mu \) by at least 1 standard deviation?

c) What value of \( \sigma \) would make \( P(22 \leq X) = 0.01 \)?
PROBLEM 5. Suppose that when the pH of a certain chemical compound is 5.00, the pH measured by a randomly selected beginning chemistry student is a random variable with mean 5.00 and standard deviation 0.2. A large batch of the compound is subdivided and a sample given to each student in a morning lab and each student in an afternoon lab. Let $\overline{X}$ = the average pH as determined by the morning students and $\overline{Y}$ = the average pH as determined by the afternoon students.

a) If pH is a normal variable and there are 25 students in each lab, compute $P(-0.1 \leq \overline{X} - \overline{Y} \leq 0.1)$. (Hint: $\overline{X} - \overline{Y}$ is a linear combination of normal variables, so is normally distributed. Compute $\mu_{\overline{X} - \overline{Y}}$ and $\sigma_{\overline{X} - \overline{Y}}$.)

b) If there are 36 students in each lab, but pH determinations are not assumed normal, calculate (approximately) $P(-0.1 \leq \overline{X} - \overline{Y} \leq 0.1)$. 