Large Deformation Diffeomorphic Metric Mapping of Diffusion Tensor Images

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Introduction

Diffusion tensor magnetic resonance imaging (DT-MRI) probes and quantifies the anisotropic diffusion of water molecules in biological tissues, making it possible to non-invasively infer the architecture of the underlying structures. The measurement at each voxel in a DT-MRI image volume is a symmetric second order tensor. Orientation of the principal eigenvector of the diffusion tensor is known to align with fiber tracts. Matching DT-MRIs is more complicated than matching scalar images since DT-MRIs contain orientational information, which is affected by the transformation. Our approach is to apply the theory of Large Deformation Diffeomorphic Metric Mapping (LDDMM) to compute a geodesic path on the manifold of diffeomorphisms connecting two DT-MRIs which transforms a template into target, computing the metric distances of the transformations required for registration as a measure of the differences between template and target, and building probability distributions describing variability of geometry of subjects in terms of the variability of these transformations.

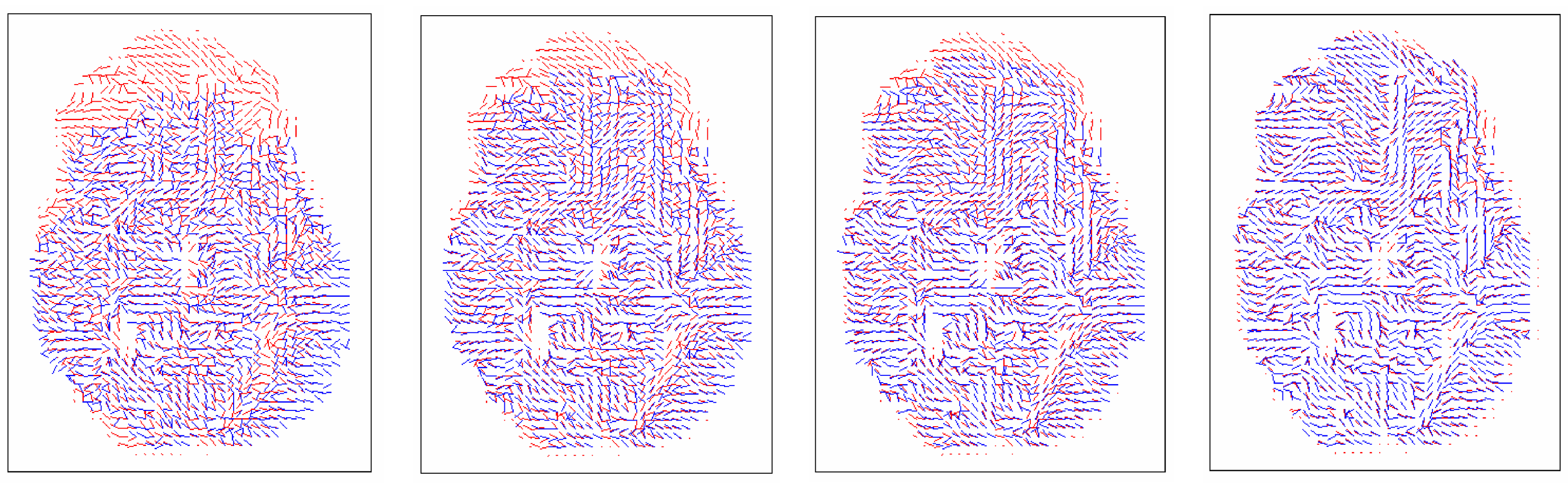


Figure 1: Panels show how the template image deforms to the target image on the geodesic path, with two images superimposed, blue the template and red the target.

Formulation

Let the background space Ω be a bounded domain in R^d and G a group of diffeomorphisms on Ω. Let the images be functions M: $\Omega \rightarrow R^d$ that associate to each point x in Ω the diffusion tensor, a symmetric second order tensor. The diffeomorphic transformations G are defined through the solutions of the nonlinear Eulerian transport equation

$$\dot{\varphi}_t = v_t(\varphi_t), \quad \varphi_0(x) = x, \quad t \in [0,1]$$

For each t, v_t is a vector field on Ω which belongs to some Hilbert space V , which is constructed using the theory of Reproducing Kernel Hilbert Spaces. And φ_t is called the flow associated to the time dependent vector field v_t. G acts on the set $\boldsymbol{I}$ of all images. For any M in $\boldsymbol{I}$ and φ in G, let λ_1 , λ_2 and λ_3 be the eigenvalues of M with $\lambda_1 \geq \lambda_2 \geq \lambda_3$, and e_1, e_2 and e_3 be the corresponding eigenvectors. The action is defined as follows[1]:

$$\varphi.M \doteq \left(\lambda_1 \hat{e}_1 \hat{e}_1^T + \lambda_2 \hat{e}_2 \hat{e}_2^T + \lambda_3 \hat{e}_3 \hat{e}_3^T\right) \circ \varphi^{-1}$$

where

$$\hat{e}_1 = \frac{D\varphi e_1}{\|D\varphi e_1\|}$$

$$\hat{e}_2 = \frac{D\varphi e_2 - \langle \hat{e}_1, D\varphi e_2 \rangle \hat{e}_1}{\sqrt{\|D\varphi e_2\|^2 - \langle \hat{e}_1, D\varphi e_2 \rangle^2}}$$

$$\hat{e}_3 = \hat{e}_1 \times \hat{e}_2$$

Given two images M_0 and M_1, the optimal matching φ between M_0 and M_1 is generated as the end-point $\varphi = \varphi_1$ of the flow. Estimation of the optimal transformation is done via the following variational problem:

$$\hat{v} = \arg\min_v \left(\int_0^1 \|v_t\|_V^2 dt + \alpha \int_\Omega dist(\varphi.M_0, M_1)^2 \, dx \right)$$

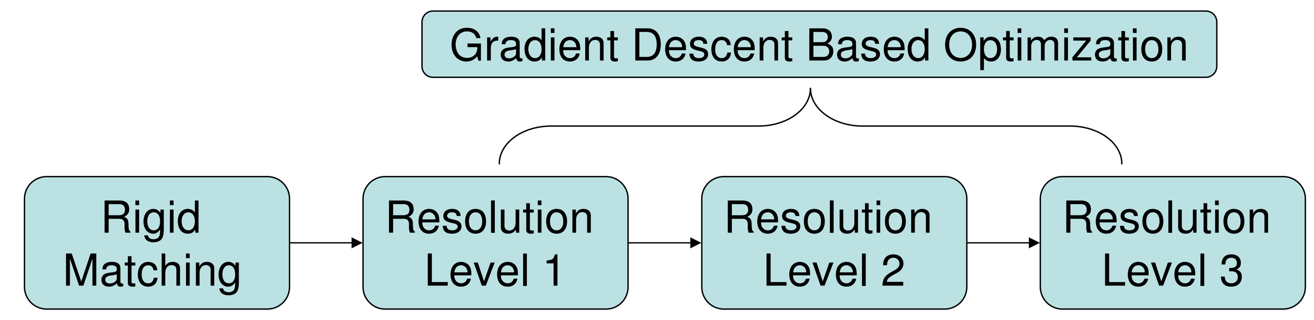
where

$$dist(\varphi.M_0, M_1) = \|\varphi.M_0 - M_1\|_F = trace\left[(\varphi.M_0 - M_1)^*(\varphi.M_0 - M_1)\right]$$

Conclusion

We have presented a method to match DT-MRIs via LDDMM of tensor fields. The optimal mapping is the endpoint of a geodesic path on the manifold of diffeomorphisms connecting two tensor fields. Finding the optimal mapping and the geodesic path is formulated as a variational problem over a vector field. A gradient descent based multi-resolution multi-kernel-width algorithm is implemented. We expect this method to be a useful tool for analysis of DT-MRIs and other images which have similar properties.

Algorithm



We calculated the first order variation of the energy function with respect to the vector field v. The variational optimization is performed in a steepest descent scheme. The step size is decided by a line search algorithm in the direction of steepest descent. To generate the map φ from the vector field v, we use a second-order semi-Lagrangian scheme to solve the transport equation about φ.

We performed a rigid transformation matching before applying the gradient descent scheme. A hierarchical multi-resolution strategy is used in our algorithm to reduce the ambiguity problem and the computation load. It is employed from coarse to fine, and results achieved on one resolution are considered as approximations for the next finer level. We generate the image pyramid by reducing the resolution from one level to the next by a factor of 2.

Experimental Results

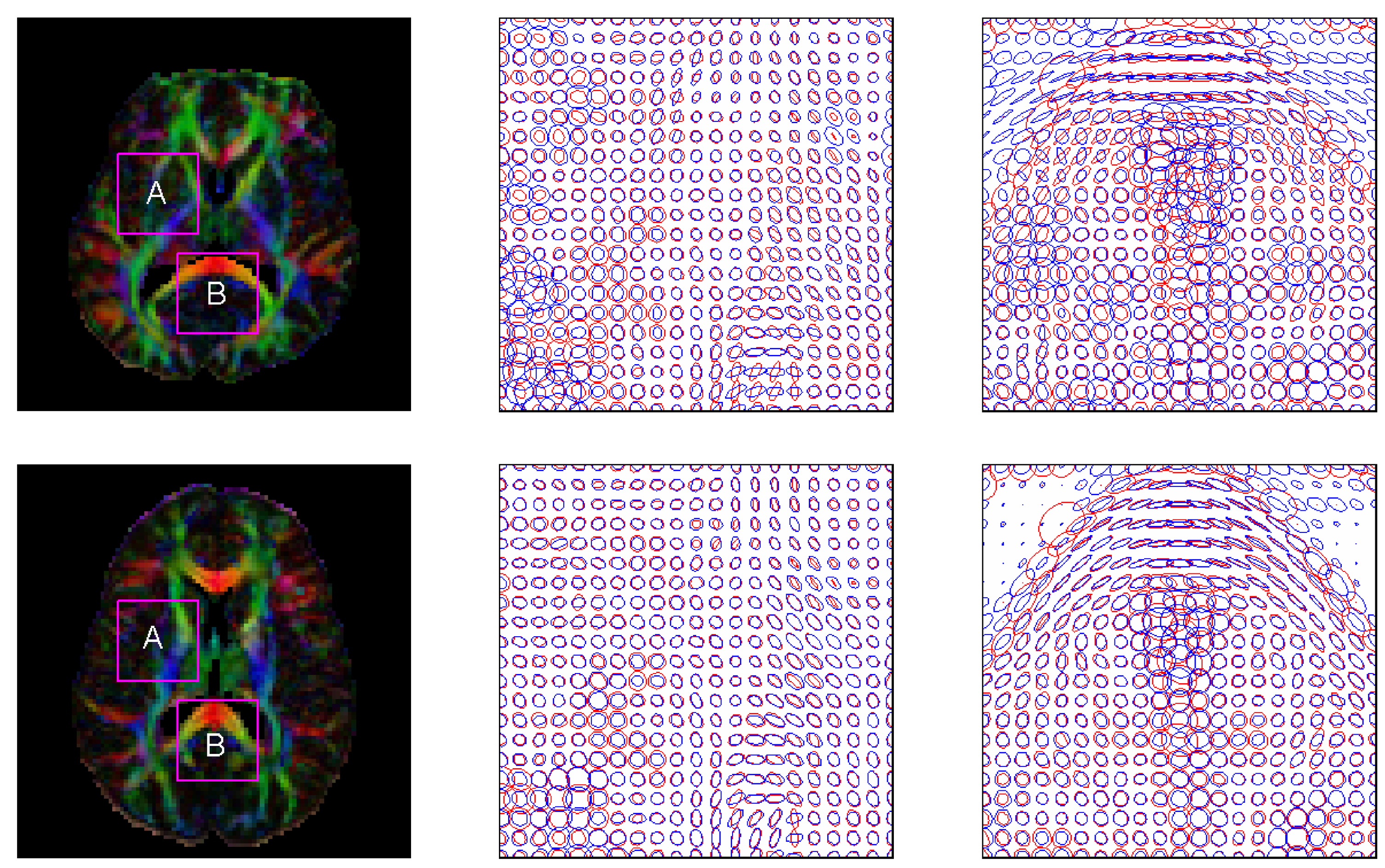


Figure 2: 3D tensor matching of two normal human brains. First column shows the fractional anisotropy weighted color-coded orientation map of slice 30 of the template and the target respectively. Second and third column show the tensor distribution of region A and region B respectively. Template and target are superimposed with blue color the template and red color the target. Top row shows the tensor distribution before matching; bottom row shows the tensor distribution after matching.

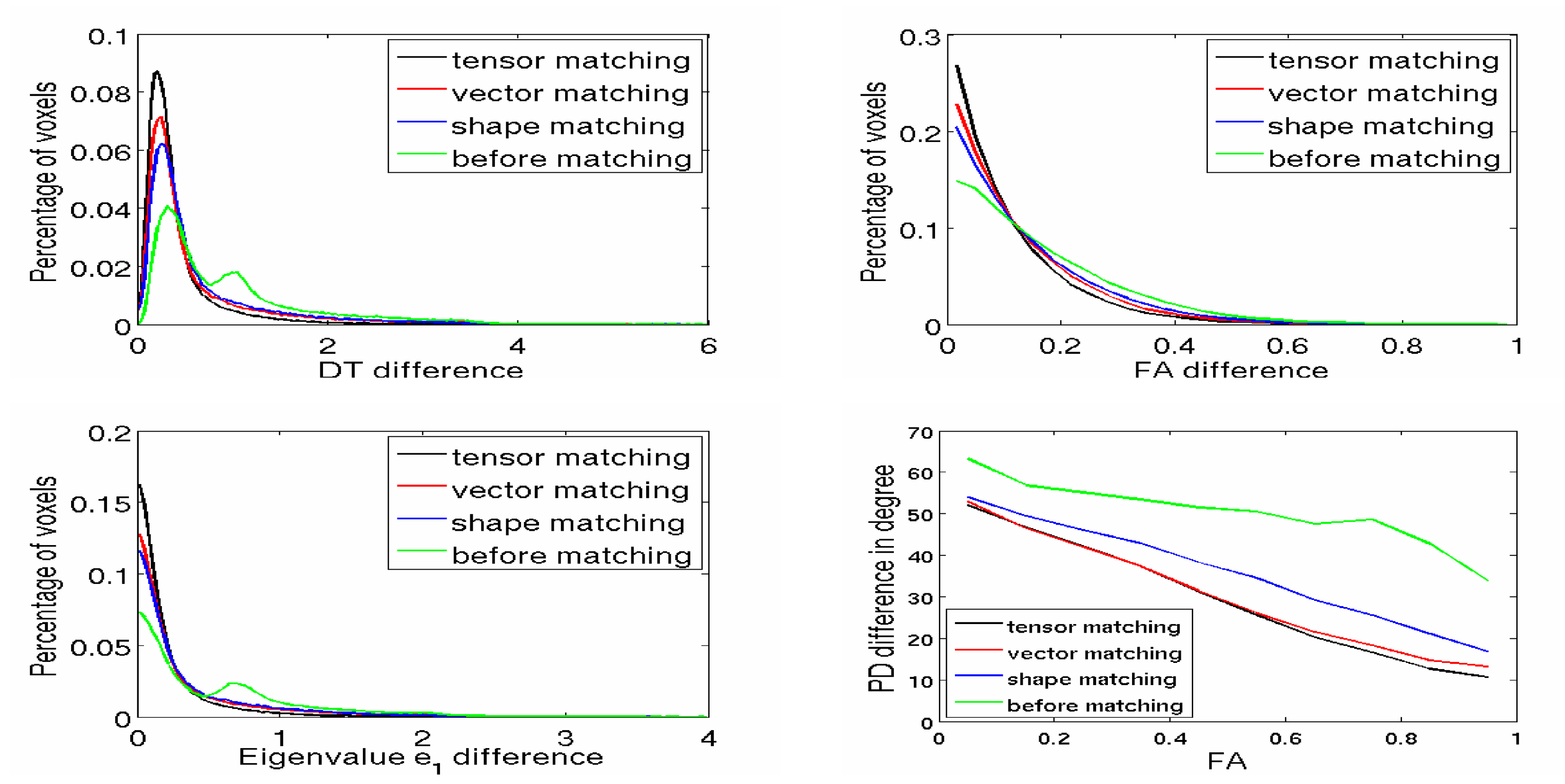


Figure 3: Comparison of the deformed template and the target for different LDDMM matching schemes. Top left panel shows the histogram of the tensor difference at each voxel. Top right panel shows the histogram of the fractional anisotropy (FA) difference at each voxel. Bottom left panel shows the mean difference between corresponding principal eigenvectors in function of FA value. Bottom right panel shows the histogram of the eigenvalue e_1 difference at each voxel.

References:
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